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NAVWEPS REPORT 8075
NOTS TP 3093
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SIMPLIFIED METHODS OF FINDING THE KILL PROBABILITIES OF CLUSTER WEAPONS AGAINST UNITARY TARGETS

Extending the Results of Navord Report 7019

by

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ABSTRACT. Similarities and relationships among the graphs of NAVORD REPORT 7019 (A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets, by Eldon L. Dunn) indicate that salvo kill probabilities can often be estimated for larger numbers of bomblets and larger ratios of delivery standard deviations to target dimensions without further machine computation, either by changing the parameter values of the original graphs according to simple rules or by using generalized graphs derived from the original ones. These methods, which essentially were developed empirically, are supported by an examination of the basic equation of NAVORD REPORT 7019 and the introduction of two appropriate approximations. (UNCLASSIFIED)

U.S. NAVAL ORDNANCE TEST STATION
China Lake, California
OCTOBER 1962

U. S. NAVAL ORDNANCE TEST STATION

AN ACTIVITY OF THE BUREAU OF NAVAL WEAPONS

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FOREWORD

Continuing interest in the possibility of using clustered-warhead weapons in non-nuclear warfare has given sustained popularity to NAVORD REPORT 7019 (A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets, by Eldon L. Dunn). Convenient methods of extending the results to larger numbers of bomblets and larger dispersions without additional machine computation have been employed recently at the U. S. Naval Ordnance Test Station. It is believed that these methods will be of interest to users of the "Handbook" and others faced with the problem of estimating the effectiveness of cluster weapons.

The work reported here was supported by WEPTASK RM-3301-001/216-1/F009-01-009.

This report was reviewed for technical adequacy by Eldon L. Dunn.

Released by
G. S. COLLADAY, Head,
Weapons Planning Group
17 September 1962

Under Authority of
WM. B. McLEAN
Technical Director

NOTS Technical Publication 3093
NAVWEPS REPORT 8075

Published by . . . . Weapons Planning Group
Collation . . . . .Cover, 10 leaves, abstract cards
First Printing . . . . 170 numbered copies
Security classification .UNCLASSIFIED

INTRODUCTION

Interest in the possibility of increasing the effectiveness of conventional weapons by employing clusters or salvos of many bomblets has been accompanied by corresponding interest in the problem of estimating the over-all kill probability of cluster weapons. An effective approach to this problem has appeared in NAVORD REPORT 7019 by Eldon L. Dunn,[1] in which the results of a parametric study of cluster-weapon kill probability against unitary targets are presented in handbook form. A unitary target is one that can be said to be destroyed (or damaged to a specified extent) as a unit; it is to be distinguished from a target or group of targets that can be said to be damaged over a fraction of its total area (area target). Only unitary targets are considered in this report.

Although the graphs in the Handbook (NAVORD REPORT 7019) are quite useful in estimating the effectiveness of proposed cluster weapons, their applicability is sometimes limited by the difficulties of interpolation and by the absence of data for more than 400 bomblets or for delivery standard deviations greater than 8 target widths. It has been observed, however, that some simple relationships appear to exist among the parameters in the Handbook within certain ranges of their values. Simple rules can be derived for extending the ranges of some variables by using "artificial" values of other variables with the graphs provided in the Handbook.

In this report, these empirically determined rules are briefly described and illustrated. Graphs based on these rules are presented to permit salvo kill probabilities to be estimated without using the Handbook, for many cases both within and beyond the scope of the Handbook graphs.

Then, in an attempt to provide insight into the basic relationships and to verify their reliability in extrapolating the Handbook data, the basic equation used in the preparation of the Handbook is examined. Two simple approximations and some changes in notation are introduced to aid

[1] U. S. Naval Ordnance Test Station. A Handbook on the Effectiveness of Cluster Weapons Against Unitary Targets, by Eldon L. Dunn. China Lake, California, NOTS, 12 January 1960. (NAVORD REPORT 7019, NOTS TP 2382)

the discussion, which generally supports the application of the empirical rules to cases where standard deviations and numbers of bomblets are relatively large. It appears that the basic equation can, for such cases, be "condensed" into a form in which the salvo kill probability is approximately a function of just two variables that correspond to those that appear in the empirically derived graphs.

In an appendix, a few minor errors in the Handbook are briefly listed for its users' convenience.

THE EMPIRICALLY DETERMINED RULES

The observations described in this section essentially make up the foundation of this report. While the mathematical discussion is important in understanding and generalizing the observed relationships, both the original inspiration and the determination of approximate limits of application have come from inspection of the Handbook graphs. The reader who has a copy of the Handbook and is interested only in some simple methods of interpolating and extrapolating the data may find this section adequate for his needs. On the other hand, the reader who does not have at hand a copy of the Handbook is encouraged to pass quickly over the many references to the Handbook, since the graphs and equations that appear later in this report can be used independently.

The problem treated in the Handbook is that of finding the salvo kill probability of a cluster weapon whose bomblets are normally distributed about a point that is itself normally distributed about the aimpoint, against a rectangular, unitary target.

The graphs in Appendix A of the Handbook show the salvo kill probability (P) as a function of the ammunition standard deviation (σ_{R_x}) for various values of delivery standard deviation (σ_{F_x}), values of probability that a hit by a bomblet is a kill (K_P), numbers of bomblets (N), and target lengths (L). The values of σ_{R_x} , σ_{F_x} , and L are given in terms of target width (W). (Specifying the x-direction standard deviations, σ_{R_x} and σ_{F_x} , also specifies the y-direction quantities, since the ratios $\sigma_{R_y}/\sigma_{R_x}$ and $\sigma_{F_y}/\sigma_{F_x}$ are fixed at 1.5 in the Handbook.) The values of σ_{R_x} that correspond to maximum values of P on each curve are referred to here as optimum values. The graphs in Appendices C and D of the Handbook show just the maximum salvo kill probability (P_o) as a function of N or of σ_{F_x} , when σ_{R_x} is always optimum.

Certain similarities among curves from these Handbook graphs become apparent when they are used extensively.[2] For example, if N is changed

[2] These similarities were first pointed out to the author by M. L. Braithwaite of the Weapons Development Department at the U. S. Naval Ordnance Test Station.

but K_p is also changed so that the product NK_p is not changed, then the value of P often is not changed. Thus, the curve for $N = 200$ and $K_p = 0.4$ from Fig. A59 of the Handbook is about the same as the curve for $N = 400$ and $K_p = 0.2$ from Fig. A60. Further, the same curve will hold for $N = 800$ and $K_p = 0.1$ or for $N = 0.05$, if σ_{F_x} and target length remain $2W$ and $3W$, respectively. (This has been verified by separate machine computation with $N = 800$ and 1600.) It appears, then, that the simplest method for extrapolating the Handbook data to larger values of N is to replace the desired values of N and K_p with "artificial" values N^* and K_p^* that appear in the Handbook and satisfy $N^*K_p^* = NK_p$.

Example 1: Find the maximum probability (P_o) for a target 20 x 60 ft with $\sigma_{F_x} = 80$ ft, $\sigma_{F_y} = 120$ ft, $K_p = 0.025$, and $N = 1600, 3200, 6400$. Since $\sigma_{F_x} = 4W$, $\sigma_{F_y}/\sigma_{F_x} = 1.5$, and the target length is $3W$, Fig. C11 in the Handbook is used with $K_p^* = 0.4$ and $N^* = 100, 200, 400$. Then $P_o = 0.35, 0.52, 0.70$.

Relationships involving other parameters are also observed, but they do not hold as generally as the one that involves just N and K_p. If σ_{F_x} is large, it can be seen that P is approximately the same if the product NK_pL is the same, where L is the target length. Compare, for example, the curve for $K_p = 0.6$ in Fig. A29 of the Handbook with the curve for $K_p = 0.1$ in Fig. A66. In each case, $\sigma_{F_x} = 4W$ and the product $NK_pL = 120$. The curves are not identical, but the values of P differ by only about 0.01. Under conditions when σ_{F_x} is large enough, then, artificial values N^*, K_p^*, L^* that can be found in the Handbook graphs can be substituted for desired values N, K_p, L if $N^*K_p^*L^* = NK_pL$.

Similarities among curves with different values of σ_{F_x} can also be observed. For example, Fig. A28 for $\sigma_{F_x} = 4W$ and $N = 100$ appears to be nearly the same as Fig. A36 for $\sigma_{F_x} = 8W$ and $N = 400$. It is necessary, however, to change the σ_{R_x} scales to scales of $\sigma_{R_x}/\sigma_{F_x}$, so that all graphs in Appendix A of the Handbook have the same horizontal scale. Since a change by some factor in σ_{F_x} often is equivalent to a change by the square of the factor in N or K_p, it appears that in many cases P is approximately the same if the quantities $NK_pL/\sigma_{F_x}^2$ and $\sigma_{R_x}/\sigma_{F_x}$ are the same. The curves for which $NK_pL/\sigma_{F_x}^2$ is the same are nearly alike if σ_{F_x} is large enough, but for smaller σ_{F_x} the curves begin to diverge for the smaller values of $\sigma_{R_x}/\sigma_{F_x}$. In general, the values of P agree within about 0.02 if σ_{R_x} is large enough that either σ_{R_x} is not less than its optimum value or σ_{R_x} and σ_{R_y} are larger than about half the corresponding target dimensions. Also, N should be larger than about 10.

Example 2: Find P for a target 20 x 60 ft with σ_{F_x} = 160 ft, K_P = 0.6, and N = 1600, if σ_{R_x} = 40 ft, 240 ft ($\sigma_{R_y}/\sigma_{R_x} = \sigma_{F_y}/\sigma_{F_x}$ = 1.5). Also find the optimum value of σ_{R_x} and the corresponding maximum probability, P_o. It is seen that σ_{F_x} = $8W$. By adopting the artificial values N^* = 400 and $\sigma_{F_x}^*$ = $4W$, Fig. A66 in the handbook can be used. For σ_{R_x} = $\frac{1}{4}\sigma_{F_x}^*$ (that is, 40 ft or $1W$), the curve for K_P = 0.6 gives P = 0.34; for σ_{R_x} = 1.5 $\sigma_{F_x}^*$ (that is, 240 ft or $6W$), P=0.74. The optimum value of σ_{R_x} is about 1.05 $\sigma_{F_x}^*$ or about 168 ft and the maximum probability is about 0.79. From separate machine computation, the results are P = 0.32 and 0.74 for σ_{R_x} = 40 ft and 240 ft, and P_o = 0.795 at σ_{R_x} = 165 ft.

If only the maximum values, P_o, are desired, the graphs in Appendices C and D of the Handbook are used with any combination of artificial values that satisfy $N^*K_P^*L^*/\sigma_{F_x}^{*2} = NK_P L/\sigma_{F_x}^2$. (The ratio $\sigma_{R_x}/\sigma_{F_x}$ always has its optimum value in this case.) The next two examples show how easily some of the examples in the Handbook are worked by this method. In both cases, σ_{F_y} = 1.5 σ_{F_x}.

Example 3: Find P_o for a target 40 x 120 ft with σ_{F_x} = 60 ft (= $1.5W$), K_P = 0.5, and N = 10. Since curves for K_P = 0.5 do not appear in the Handbook, K_P^* = 0.6 is used with $\sigma_{F_x}^*$ = $1.64W$. The value of P_o is then 0.35, read from Fig. D8 of the Handbook. (The linear interpolation suggested on page 7 of the Handbook gives 0.34 for P_o.)

Example 4: Find P_o for a target 10 x 20 ft with σ_{F_x} = 30 ft (= $3W$), K_P = 0.45, and N = 120. In this case, none of the parameters has a value that appears in the Handbook. Letting K_P^* = 0.4, L^* = 3, and $\sigma_{F_x}^*$ = $4W$, N^* is computed from $N^*(0.4)(3)/16 = (120)(0.45)(2)/9$. Then N^* = 160, and P_o = 0.46 from Fig. C11 of the Handbook. A separate machine computation also gives 0.46 for P_o.

It is easily seen that this method is less complicated and more accurate than the interpolation procedure by which the same problem as Example 4 is attacked on pp. 7-9 of the Handbook.

If the ratio $\sigma_{F_y}/\sigma_{F_x}$ is not 1.5, the formula for artificial values can be written in the form

$$(1) \qquad N^*K_P^*L^*/\sigma_{F_x}^{*2} = 1.5\, NK_P L/\sigma_{F_x}\sigma_{F_y}$$

When finding values of P_o, this is an extension of the "simplified method" described on pp. 9-13 of the Handbook. If $N^* = N$ and $K_p^* = K_p$, (1) above is the same as Eq. (2) on p. 11 of the Handbook, since K in the Handbook is L^* and csc δ in the Handbook is $\sigma_{F_y}/\sigma_{F_x}$. Of course, when finding P from the curves in Appendix A of the Handbook, the condition

(2) $\sigma_{R_x}^*/\sigma_{F_x}^* = \sigma_{R_x}/\sigma_{F_x} = \sigma_{R_y}/\sigma_{F_y}$ must also be observed.

Example 5: Find P_o for the same conditions as Example 4, except $\sigma_{F_y}/\sigma_{F_x} = 1$. Then N^* is 240 and the curve for $K_p = 0.4$ from Fig. C11 of the Handbook gives $P_o = 0.57$. This is also the correct (machine-computed) value. Similarly, if $\sigma_{F_y}/\sigma_{F_x} = 2$, N^* is the same as the original N, 120, and P_o from the same curve is 0.39.

Values of P_o from the Handbook graphs are accurate within about 0.01 if N is greater than 10, σ_{F_x} is greater than $\frac{1}{2}W$, and σ_{F_y} is greater than $\frac{1}{2}L$. Values of P are generally accurate within about 0.02 if N is greater than 10 and either $\sigma_{R_x}/\sigma_{F_x}$ is not less than optimum or σ_{R_x} and σ_{R_y} are greater than about half the corresponding target dimensions.

METHODS FOR WHICH THE HANDBOOK IS NOT REQUIRED

The methods just discussed for interpolation and extrapolation of the Handbook graphs suggested the possibility of "condensing" some of the data of the Handbook into a few curves. Pursuing this idea, if P_o is approximately the same when the quantity $NK_p L/\sigma_{F_x}^2$ is the same, it seems natural to adopt a single new parameter Z, where

(3) $$Z = NK_p L/\sigma_{F_x}^2$$

and plot a single curve showing P_o as a function of Z. Such a curve appears here in Fig. 1. Values from the Handbook curves lie very close to this curve if N is larger than about 10 and σ_{F_x} is larger than both $W/2$ and $L/3$. In general, of course, since the ratio $\sigma_{F_y}/\sigma_{F_x} = 1.5$ is included in the Handbook data, the equation for Z should include both σ_{F_y} and σ_{F_x}. Also, the values of σ_{F_y}, σ_{F_x}, and L need not be expressed in target widths (W) if W appears in the numerator of the formula. The product LW is just the target area, T, so the general form of (3) is

(4) $$Z = 1.5\, NK_p T/\sigma_{F_x}\sigma_{F_y}$$

When P_o is read from Fig. 1 using Z computed from (4), it is accurate within about 0.01 if N is greater than 10 and σ_{F_x} and σ_{F_y} are greater than about half the corresponding target dimensions.

Examples: For the previous examples, P_0 is easily found from Fig. 1 as follows:

• In example 1, for N = 1600, 3200, and 6400, the values of Z are 7.5, 15, and 30. From Fig. 1, P_0 = 0.35, 0.52, and 0.70, in agreement with the values from the Handbook.

• In example 2, Z = 45, and P_0 from Fig. 1 is 0.80. The separate machine computation gave 0.795.

• In example 3, the value of N is rather low for this method; still, the value Z = 6.67 gives P_0 = 0.33 from Fig. 1. The value from Fig. D8 of the Handbook is 0.35.

• In example 4, Z = 12, and P_0 = 0.47 from Fig. 1. A separate machine computation gives 0.46.

• In example 5, for $\sigma_{F_y}/\sigma_{F_x}$ = 1 and 2, Z = 18 and 9. The values of P_0 from Fig. 1 are 0.57 and 0.39, in agreement with the previous results.

Although Fig. 1 is very useful for estimating values of P_0, it is not applicable if other than optimum values of σ_{R_x} and σ_{R_y}, the ammunition standard deviations, are considered. Even when finding P_0, it does not indicate what the optimum values of σ_{R_x} and σ_{R_y} are. It appears, however, that the salvo kill probability, P, may be regarded in many cases as a function of just two variables, Z and M, where Z is defined in (4) and M is the ratio of ammunition standard deviation to delivery standard deviation. That is, if

$$(5) \qquad M = \sigma_{R_x}/\sigma_{F_x} = \sigma_{R_y}/\sigma_{F_y}$$

P can be plotted as a function of M for various values of Z. Such a family of curves appears here as Fig. 2. In general, each curve is adapted from one in the Handbook for which Z has the desired value and the target dimensions are small with respect to the standard deviations. Values of P estimated from Fig. 2 are accurate within about 0.03 if N is greater than 10 and either M is not less than optimum or σ_{R_x} and σ_{R_y} are greater than about half the corresponding target dimension. The curves are also useful for estimating optimum values of M. It appears that the ammunition standard deviations are required to be about the same size as the delivery standard deviations (M is near 1) if high values of salvo kill probability are to be efficiently achieved.

Examples: In several of the examples treated earlier, Fig. 2 is now used to estimate optimum values of σ_{R_x} and σ_{R_y}, as follows:

. In example 1, optimum values of M for Z = 7.5, 15, and 30 are approximately 0.65, 0.80, and 1.0, respectively. Since σ_{F_x} is 80 ft, the optimum values of σ_{R_x} are 52 ft, 64 ft, and 80 ft; the corresponding optimum values of σ_{R_y} are 78 ft, 96 ft, and 120 ft.

. In example 3, where Z = 6.67, the optimum value of M is about 0.62. For σ_{F_x} of 60 ft, optimum values of σ_{R_x} and σ_{R_y} are 37 ft and 56 ft, respectively.

. In examples 4 and 5, the optimum ammunition dispersions are easily determined in the same manner:

(Target 10 x 20 ft, σ_{F_x} = 30 ft, N = 120, K_p = 0.45)

$\sigma_{F_y}/\sigma_{F_x}$	Z	P_o	Optimum Values of		
			M	σ_{R_x}, ft	σ_{R_y}, ft
1.5	12	0.47	0.78	23	35
1.0	18	0.57	0.84	25	25
2.0	9	0.39	0.7	21	42

It has been assumed here that the ratio of σ_{R_y} to σ_{F_y} is always the same as the ratio of σ_{R_x} to σ_{F_x}. It seems reasonable to suppose that the same characteristics of an attack that produce an elliptical distribution of delivery error will produce a similarly elliptical distribution of bomblet impacts. The same assumption is made implicitly in the Handbook, and will be introduced again in the next section to simplify the mathematical derivation. It is this common ratio, M, that is optimized when the maximum probability for a given value of Z is found. This ratio (but not necessarily the separate components σ_{R_x} and σ_{R_y}) is also optimized in the Handbook when σ_{R_x} is optimized.

While the formulas and figures just presented make possible the estimation of values of salvo kill probability without further use of high-speed digital computers, and while the estimated values agree well with the results of special machine computations for larger numbers of bomblets and larger standard deviations, a degree of mathematical justification seems desirable. In the next section, the basic equation used in the preparation of the Handbook is briefly examined in the light of the empirical rules and methods already described.

DISCUSSION OF THE BASIC EQUATION

As described on pp. 2-4 of the Handbook, the problem is to find the probability, P, that at least one round of a salvo (cluster) will kill the target. If the aim point of the fire-control system and the center of the target are both at the origin, the basic equation on p. 2 of the Handbook may be written as

$$(6)\quad P = \frac{1}{2\pi\,\sigma_{F_x}\,\sigma_{F_y}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} PL \; exp\left[-\frac{1}{2}\left(x^2/\sigma_{F_x}^{\,2} + y^2/\sigma_{F_y}^{\,2}\right)\right] dx\,dy$$

Here, PL is the probability that at least one round of a salvo whose projected impact point is (x, y) will kill the target. That is,

$$(7)\qquad PL = 1 - \left[1 - p\,(x, y)\right]^N$$

where $p\,(x\,,\,y\,)$ is the probability that a particular round of the salvo will kill the target. Since the rounds are normally distributed about the projected impact point,

$$(8)\qquad p\,(x, y) = \frac{K_p}{2\pi}\int_A^B exp\left(-\frac{1}{2}u^2\right)du \int_C^D exp\left(-\frac{1}{2}v^2\right)dv$$

where

$$A = \left(-x - \tfrac{1}{2}XL\right)/\sigma_{R_x}$$
$$B = \left(-x + \tfrac{1}{2}XL\right)/\sigma_{R_x}$$
$$C = \left(-y - \tfrac{1}{2}YL\right)/\sigma_{R_y}$$
$$D = \left(-y + \tfrac{1}{2}YL\right)/\sigma_{R_y}$$

As in the Handbook, XL and YL are the dimensions of the target, N is the number of rounds (bomblets), K_p is the probability that a hit is a kill, σ_{F_x} and σ_{F_y} are the delivery standard deviations, and σ_{R_x} and σ_{R_y} are the ammunition (round) standard deviations.

It is comparatively easy to show that a change in the number of rounds can be approximately compensated in many cases by an inverse change in the probability that a hit by a bomblet is a kill. If $p(\,x\,,\,y\,)$ is written as $K_p\,P_h$, (2) becomes

$$(9)\qquad PL = 1 - \left(1 - K_p\,P_h\right)^N$$

If N is changed to N^* and K_p to K_p^*, where $N^* = cN$ and $K_p^* = K_p/c$ so that $N^* K_p^* = N K_p$, the equation becomes

$$(10)\qquad PL^* = 1 - \left(1 - K_p P_h/c\right)^{cN}$$

It is necessary to show that PL^* is relatively insensitive to changes in the factor c for parameter values that are of interest.

If $p(x, y)$ is small, the probability of no hits, $(1 - p)^N$, may be accurately approximated by the Poisson form, e^{-pN}, in which the factor c cancels out completely. Then PL becomes approximately $1-e^{-pN}$. Even if P is not small enough for PL to be independent of c, the relative change in PL with c is not large if N is not too small. Some sample values of PL for various values of c are as follows:

c	cN	$1-(1-p/c)^{cN}$ (for $N=10$)				
	p =	.025	.05	.10	.20	.40
1	10	.224	.401	.651	.893	.994
2	20	.222	.397	.642	.878	.988
4	40	.221	.395	.637	.872	.985
8	80	.221	.394	.634	.868	.983
$1-e^{-pN}$		.221	.393	.632	.865	.982

Since the primary objective is to extend the Handbook data to larger values of N, it appears that the use of artificial values N^* and K_p^* with the Handbook graphs, such that $N^*K_p^* = NK_p$, is a reliable technique. This conclusion is supported, as described earlier, by examination of the Handbook graphs and the results of special machine computations.

The first of the two approximations that will be introduced to change the form of the basic equation is the substitution of the Poisson form, e^{-pN}, suggested above for $(1 - p)^N$ in (7). This approximation is assumed to be acceptable if N is larger than about 10 and p is small. Here, of course, $p(x, y)$ is a variable defined by (3), but it is almost always small. It can be large only if K_p is near 1 and the target dimensions are large with respect to the ammunition standard deviations, as shown by (8). Since the second approximation will require that the ammunition standard deviations be large compared to the target dimensions, it may be assumed for this discussion that $p(x, y)$ will be small even if K_p is near 1.

For convenience in the following discussion, (8) is written as

(11) $$p(x,y) = K_p I_1(x)\, I_2(y)/2\pi$$

where $$I_1(x) = \int_A^B exp\left(-\tfrac{1}{2}u^2\right) du, \quad I_2(y) = \int_C^D exp\left(-\tfrac{1}{2}v^2\right) dv$$

Substitution of $p(x, y)$ from (11) into (7) and replacement of $(1 - p)^N$ by e^{-Np} then give

(12) $$PL = 1 - exp\left(-NK_p I_1 I_2/2\pi\right)$$

Also, before introducing the second approximation, a simple change of variables is convenient:

(13) $$x/\sigma_{F_x} = X, \quad y/\sigma_{F_y} = Y$$

In terms of X and Y, the original expression, (6), becomes

(14) $$P = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} PL \exp\left[-\frac{1}{2}(X^2+Y^2)\right] dX\,dY$$

where PL is, of course, expressed in terms of X and Y. Further, as suggested by the comparison of curves from the Handbook, the ratios of ammunition standard deviations to delivery standard deviations are introduced as parameters in place of the ammunition standard deviations themselves:

(15) $$m = \sigma_{R_x}/\sigma_{F_x}, \quad n = \sigma_{R_y}/\sigma_{F_y}$$

Then the limits of integration in (8) may be written in terms of m and n and the new variables X and Y as follows:

$$A = -X/m - XL/2m\,\sigma_{F_x}, \qquad C = -Y/n - YL/2n\,\sigma_{F_y}$$

$$B = -X/m + XL/2m\,\sigma_{F_x}, \qquad D = -Y/n + YL/2n\,\sigma_{F_y}$$

For the second approximation, it is assumed that the target dimensions, XL and YL, are small with respect to the ammunition standard deviations, σ_{R_x} and σ_{R_y}. The integrals in (8) may then be removed by the use of

$$\int_A^B f(u)\,du \doteq (B-A)\,f\left(\tfrac{1}{2}A + \tfrac{1}{2}B\right)$$

Then, in terms of X, Y, m, and n, the integrals I_1 and I_2 in (11) are written

$$I_1 \doteq (XL/m\,\sigma_{F_x})\exp\left(-\tfrac{1}{2}X^2/m^2\right)$$

$$I_2 \doteq (YL/n\,\sigma_{F_y})\exp\left(-\tfrac{1}{2}Y^2/n^2\right)$$

Substitution of these into (12) gives

(16) $$PL(X,Y) \doteq 1-\exp\left[-(NK_p T/2\pi\, mn\,\sigma_{F_x}\sigma_{F_y})\exp\left(-\tfrac{1}{2}X^2/m^2-\tfrac{1}{2}Y^2/n^2\right)\right]$$

where T is the target area $XL\,YL$. As suggested by the observations of the Handbook data, PL can now be written in terms of the parameter

Z, where

$$Z = 1.5\, N K_p\, T/\sigma_{F_x}\sigma_{F_y}$$

Also, it is both convenient and reasonable, as discussed previously, to assume that $m = n$. The common ratio can then be denoted, as before, by M; that is,

$$M = \sigma_{R_x}/\sigma_{F_x} = \sigma_{R_y}/\sigma_{F_y}$$

In terms of Z and M, then, the expression for PL is

$$(17) \quad PL(X,Y) \doteq 1 - exp\left[-(Z/3\pi M^2)\, exp\left(-\tfrac{1}{2}X^2/M^2 - \tfrac{1}{2}Y^2/M^2\right)\right]$$

Since this is the only place where any parameters are involved when this expression for PL is substituted into (14), the salvo kill probability, P, may be regarded as a function of just Z and M when the two approximations are valid. The first approximation is valid if N is large enough and the second approximation is valid if XL/σ_{R_x} and YL/σ_{R_y} are small enough. These are essentially the same conditions that were derived by inspection of the graphs in the Handbook.

When the approximate expression for PL from (17) is substituted into (14), it is convenient to change variables to polar coordinates (R, θ), where $R^2 = X^2 + Y^2$. Integration of the angular variable θ then cancels the factor $1/2\pi$ in (14), giving

$$(18) \quad P = 1 - \int_0^\infty exp\left[-(Z/3\pi M^2)\, exp(-R^2/2M^2)\right] exp\left(-\tfrac{1}{2}R^2\right) R\, dR$$

It is convenient to make another change of variable and two more changes in notation:

$$w = exp\left(-\tfrac{1}{2}R^2\right), \quad h = 1/M^2, \quad s = Z/3\pi$$

Then the approximate expression for the salvo kill probability is

$$(19) \quad P \doteq 1 - \int_0^1 exp\left(-s\, h\, w^h\right) dw$$

If desired, this compact form of the basic equation could be used in a computation program to find values of P for cases in which the approximations are valid. There is actually no reason for doing so, since exact values of P have been computed for the Handbook. In a few special cases, however, the integral in (19) assumes familiar forms and values of P can be easily calculated without machine computation. If

$h = 1$, for example, the integral becomes a simple exponential form, giving

$$(20) \quad P(h=1) = 1 - (1/s)(1 - e^{-s})$$

If $h = 2$, the integral can be expressed in terms of the standard normal probability function. In this case,

$$(21) \quad P(h=2) = 1 - (\pi/2s)^{\frac{1}{2}} \phi(2s^{\frac{1}{2}})$$

where $$\phi(a) = (1/2\pi)^{\frac{1}{2}} \int_0^a \exp(-\tfrac{1}{2}t^2)\,dt$$

Curves of P as a function of Z corresponding to these two equations are presented in Fig. 3; values for $M = 1$ were computed from (20) and those for $M = 0.707$ from (21). Values from these curves agree well with those from the Handbook in cases where the approximations might be expected to be applicable. The curves may be useful to the reader who wishes to interpolate among the curves of Fig. 2.

If the value of h is not 1 or 2, the integral in (19) can still be evaluated approximately by use of Simpson's rule, although it may be desirable to use different interval sizes in different parts of the range of integration. If values of P for just a few sets of values of M and Z are desired, the amount of hand computation required is not prohibitive. Values computed in this way directly from (19) have been used to plot curve a̲ of Fig. 4, which shows P as a function of M for $Z = 30$.

The other curves in Fig. 4 are taken from the Handbook and are all for combinations of parameter values for which $Z = 30$. The curves show the effect of larger targets or smaller standard deviations. Curve b̲, for which the standard deviations are rather large with respect to the target dimensions, is quite close to curve a̲, the graph of (19) with $Z = 30$, but the other curves diverge increasingly from a̲ and b̲ at the lower values of M. This is to be expected, since if the delivery standard deviations are not large enough with respect to the target dimensions a small value of M means that the ammunition standard deviations are not large enough for the second approximation to be useful.

It appears from Fig. 4 (and also from the Handbook graphs) that the effects of larger targets or smaller delivery standard deviations are to increase the kill probability at lower values of the ratio M and to decrease the optimum value of M somewhat. There is relatively little change in the maximum value of kill probability, which suggests that a graph such as Fig. 1 should be reliable except when the delivery standard deviations are quite small with respect to the target dimensions. Com-

parison of values from Fig. 1 and from the Handbook tends to support this conclusion.

While Fig. 1 was plotted from Handbook data, presumably it is also a graph of the function of z obtained by maximizing (19) with respect to h. An expression for this rather complicated function of z has not been derived, but one that seems to approximate it rather well is

$$(22) \quad P_o = \left[1 - \exp\left[-(z/3\pi)^{\frac{1}{2}}\right]\right]^2$$

Actually, this expression for the maximum salvo kill probability is suggested by considering a somewhat different problem, in which a uniform distribution of bomblets over an area rather than a two-dimensional normal distribution is assumed.[3] As indicated by the apparent similarity of the functions described by Fig. 1 and by (22), the maximum salvo kill probability obtained when the distribution of bomblets is optimized often seems to be quite independent of the particular type of distribution assumed for the bomblets.

[3] This problem is described in Appendix B of the following: U. S. Naval Ordnance Test Station. Tactical Use and Characteristics of Effective Free-Fall Ordnance (U), by Roy Dale Cole, M.E. O'Neill, M.R. O'Neill, and Richard B. Seeley. China Lake, Calif., NOTS, 17 October 1961. (NAVWEPS Report 7801, NOTS TP 2797), SECRET/NOFORN.

SUMMARY

The data presented in NAVORD REPORT 7019 (the "Handbook") on the kill probability of cluster weapons against unitary targets appear to include relationships that allow the ranges of values of parameters to be extended beyond the Handbook limits. It is especially in the data for larger numbers of bomblets and larger ratios of standard deviations to target dimensions that the relationships are observed.

The salvo kill probability, P, may be regarded as a function of the product NK_p, if N is larger than about 10. (N is the number of bomblets and K_p is the probability that a bomblet hit is a kill.) This means that if N is beyond the range of Handbook values, P may be determined by using the Handbook graphs with "artificial" values N^* and K_p^* such that $N^* K_p^* = NK_p$.

Further, if the ammunition standard deviations, σ_{R_x} and σ_{R_y}, are each larger than about half the corresponding target dimensions, the salvo kill probability may be regarded as a function of just two parameters Z and M, where

$$Z = 1.5\, NK_p T / \sigma_{F_x} \sigma_{F_y}, \qquad M = \sigma_{R_x}/\sigma_{F_x} = \sigma_{R_y}/\sigma_{F_y}$$

(The factor 1.5 in Z corresponds to the fixed ratio of σ_{F_y} to σ_{F_x} used in the Handbook. σ_{F_x} and σ_{F_y} are the delivery standard deviations and T is the target area.) Interpolation or extrapolation of the Handbook data is thus easily accomplished by choosing artificial values that leave Z and M unchanged but that are within the Handbook's ranges of variables.

This approach works especially well if values of the maximum salvo kill probability, P_o, are desired. Also, since P_o is a function of Z only, the functional relationship can be shown on a graph, such as Fig.1 of this report. Values of P_o from this graph appear to be accurate within about 0.01 if N is greater than about 10 and σ_{F_x} and σ_{F_y} are greater than about half the corresponding target dimensions.

A graph, such as Fig. 2, can also be constructed to show P as a function of the two variables, Z and M. Values of P from this graph seem to be accurate within a few hundredths if the ammunition dispersions are such that either M is not less than optimum or σ_{R_x} and σ_{R_y} are greater than about half the corresponding target dimensions. The importance of using optimum values of M (that is, of incorporating optimum ammunition dispersions into the weapon) is also shown by these curves. If a cluster weapon is to have a relatively high salvo kill probability the desired ammunition standard deviations should be about as large as the expected delivery standard deviations.

These empirically derived techniques are generally supported when the basic equation used in the preparation of the Handbook is examined. Two approximations that might be expected to hold if the number of bomblets is large enough and the ratios of target dimensions to standard deviations are small enough result in a rather compact form of the basic equation.

It is hoped that the methods described in this report will be helpful when the effectiveness of salvo-delivered and clustered-bomblet weapons is to be estimated. It is possible to take account of rather large variations in parameter values without requiring additional long (costly) use of high-speed computing equipment.

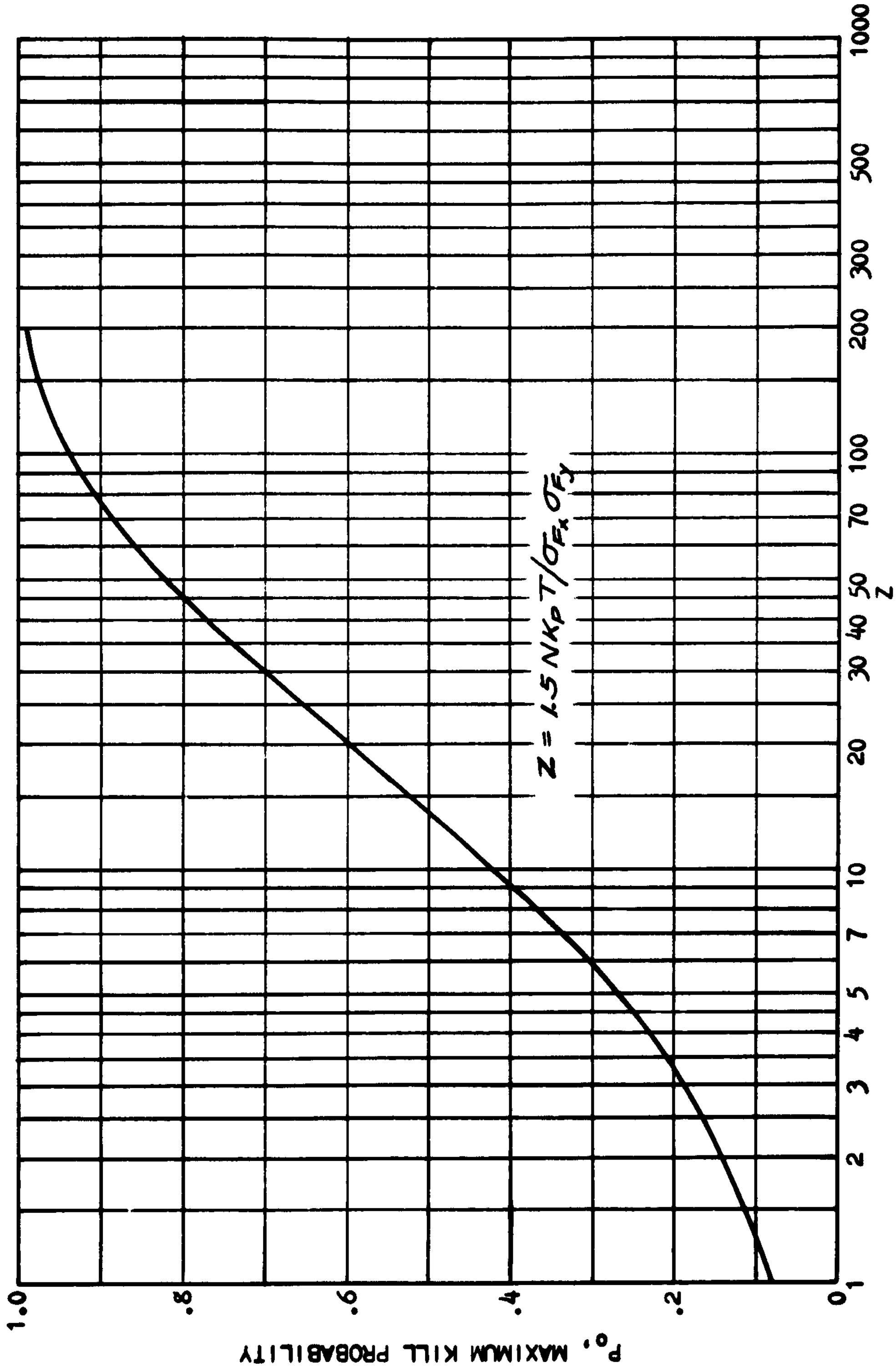

FIG. 1. Maximum Kill Probability Vs. Z. (Optimum Ammunition Dispersion)

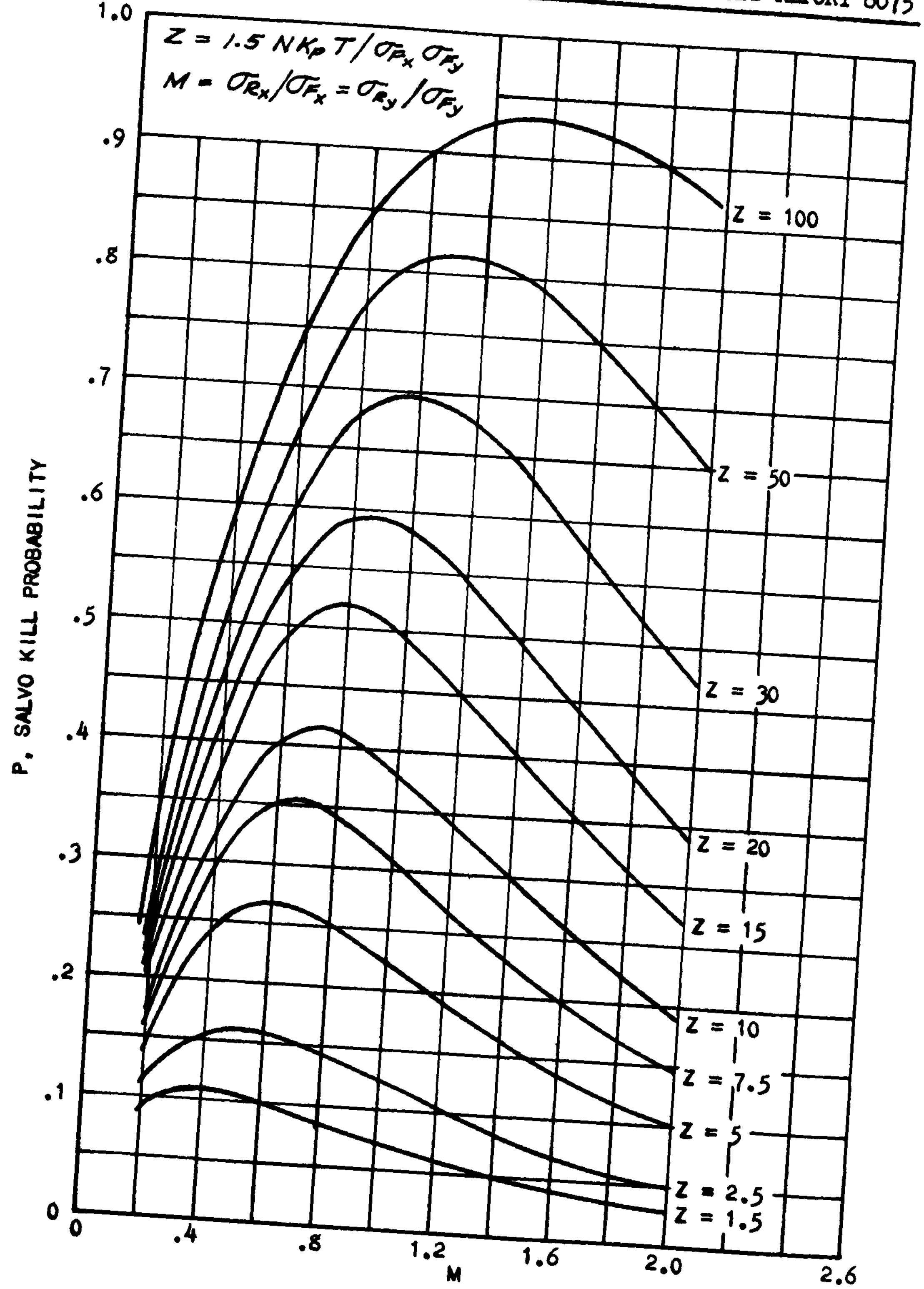

FIG. 2. Curves of Salvo Kill Probability Vs. the Ratio M for Various Volues of Z.

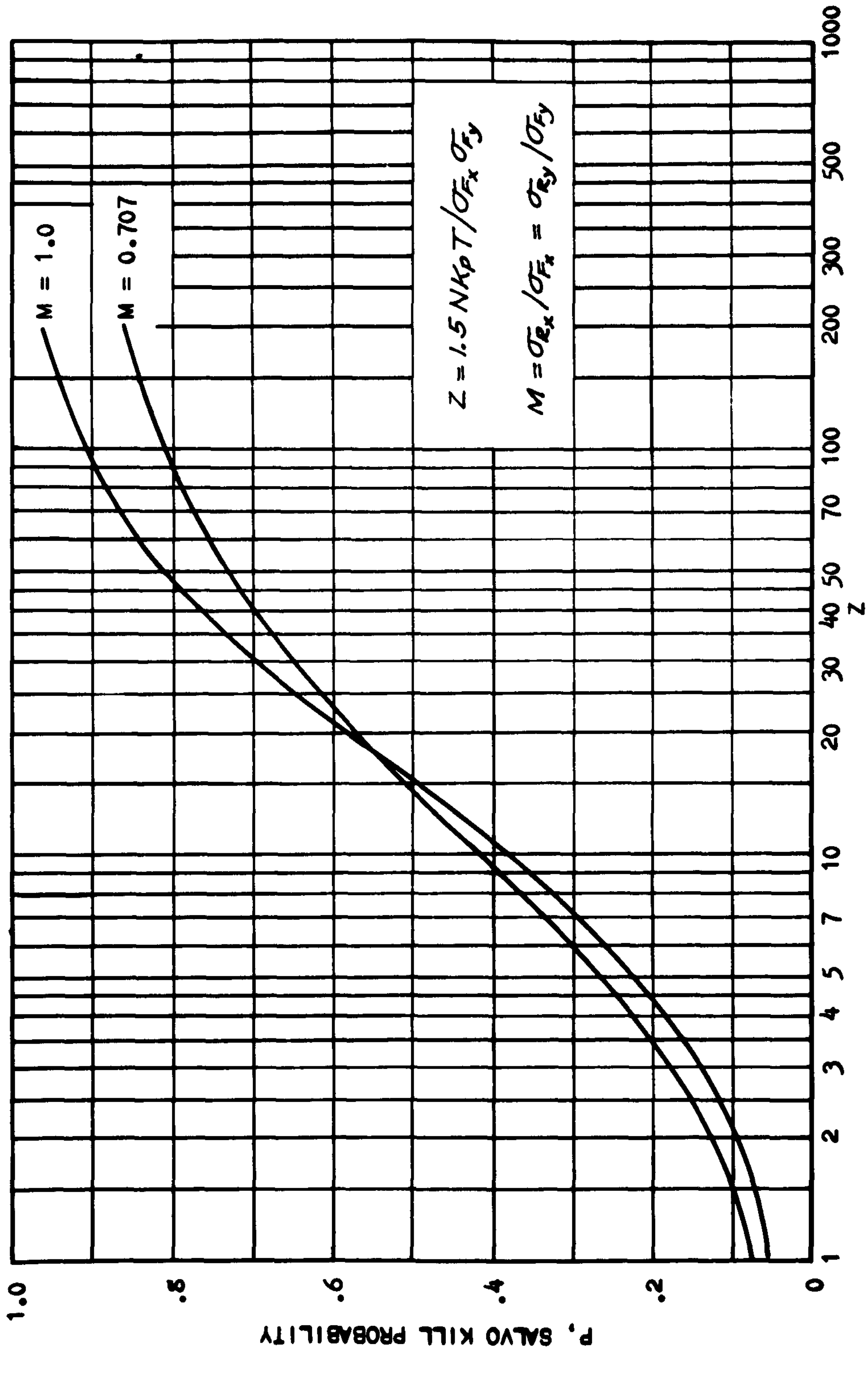

FIG. 3. Two Curves of Salvo Kill Probability Vs. Z Computed Directly from the Approximate Equation.

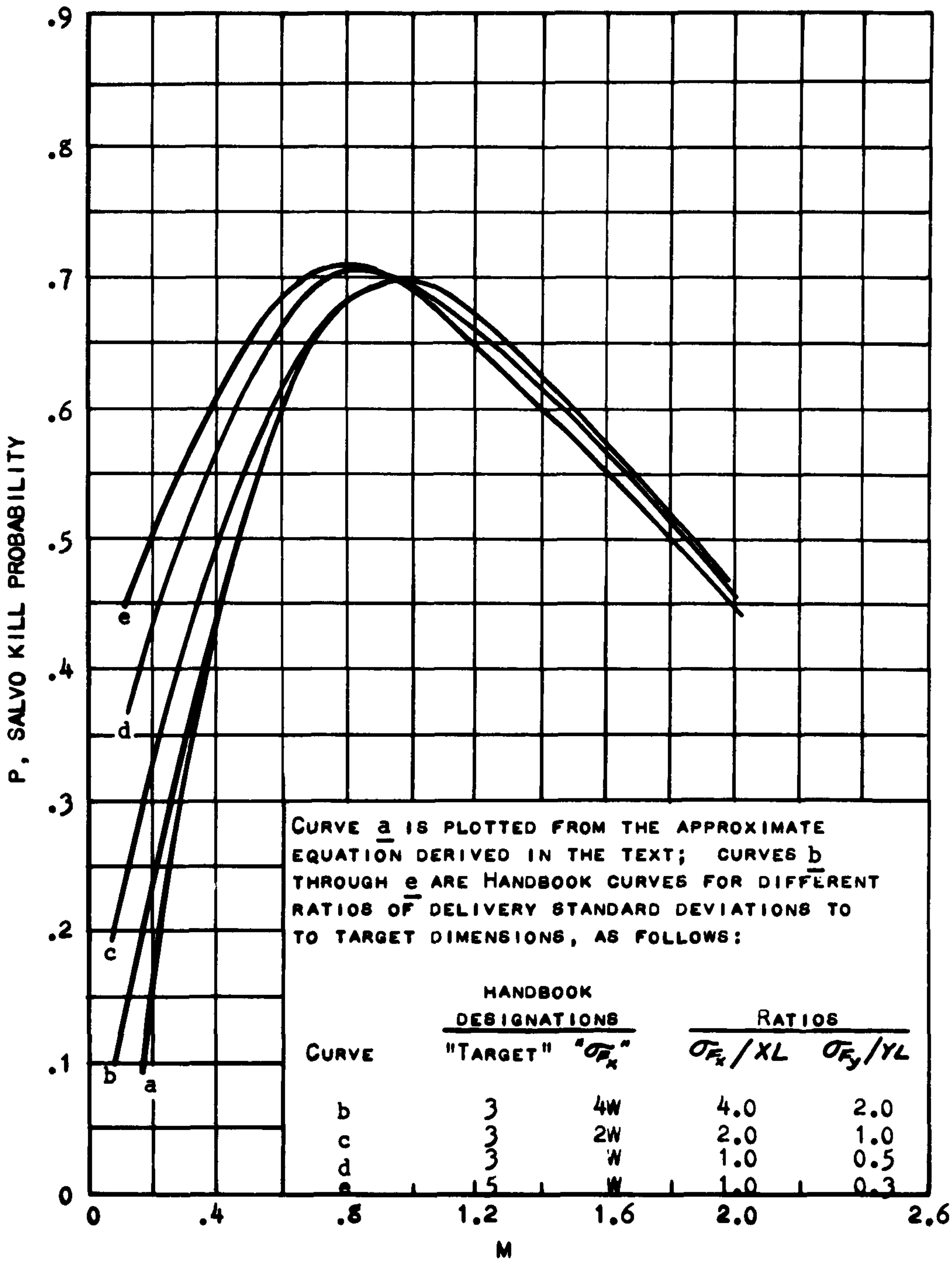

FIG. 4. Comparison of Curves of P Vs. M Having Same Value of Z but Different Ratios of Delivery Standard Deviations to Target Dimensions.

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